

# Readers' Forum

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## Comment on "Finite-Element Solution of the Supersonic Flutter of Conical Shells"

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ON the recent work by Bismarck-Nasr and Costa-Savio,<sup>1</sup> it is pertinent to make some comment referring to our previous results. A similar study<sup>2-5</sup> has been performed by us more comprehensively. Using a finite-element model (FEM), Bismarck-Nasr and Costa-Savio obtained the dynamic pressure parameter  $\gamma_{cr}$  of the flutter boundary for a conical shell and compared it with the results of Dixon and Hudson.<sup>6</sup> Calculations on the same model, conducted in a manner similar to Ref. 1, were also carried out by the present authors.<sup>4</sup> Differences between the analysis of Ref. 1 and that of Ref. 4 are illustrated in Table 1.

We cite the results from Ref. 4 in Table 2 which shows the values of the critical dynamic pressure parameter  $\gamma$  of the conical shell for the case with no aerodynamic damping. To this, Ref. 1 gives  $\gamma_{cr} = 670$  ( $n = 6$ ) in the ten-element solution. Our analysis indicates that the combined effect of the differences between the two solutions is to change the critical dynamic pressure for the ten-element solution by less than 5%. Although the importance of differences 1 and 2 in Table 1 is emphasized in Ref. 1, these differences, along with the influence of 4, do not alter the essential property of the vibration of which mode has laterally dominant deflection and several circumferential nodes. This can also be seen in natural frequencies shown in Table 3 which compares the results for 120 deg conical frustum with those in Ref. 7.

Besides, Table 2 indicates a more important factor, i.e., convergence in accordance with the refinement of finite elements. The solutions with 20 elements agree well with the results from Galerkin's method.<sup>6</sup> It is obvious from Table 2 that ten elements are insufficient for the calculation of critical values of flutter speed, notwithstanding that the convergence was examined in Ref. 1 in free vibrations. The insufficiency is attributed to the fact that the flutter occurs as a result of streamwise coupling of the first and second natural modes, which necessitates a check for convergence on the second mode as well. The authors recommend at least 20 elements for this kind of calculation. Finally, the condensation technique is also recommended from the viewpoint of efficiency.

Table 1 Comparison of analyses

Parameter	Ref. 1	Ref. 4
1) Shell theory	Novozhilov	Donnell
2) Inplane inertia	Included	Neglected
3) Boundary conditions	$u = v = w = 0$	$u = w = 0$
4) FEM degrees of freedom	Full	Condensed
5) Poisson's ratio	0.29	0.30

Table 2 Dynamic pressure parameter  $\gamma$ 

$n$	Ref. 6 Galerkin's method 12 terms	5 elements (4 DOF <sup>a</sup> )	10 elements (9 DOF)	15 elements (14 DOF)	20 elements (19 DOF)	20 elements (10 DOF)
4	1074	> 1380	1300	1130	1080	1082
5	590	> 1380	738	636	605	609
6	607	1330	700	636	616	625
7	652	> 1380	821	710	675	695

<sup>a</sup>DOF, degrees of freedom.

Table 3 Natural frequencies of a 120 deg conical frustum, Hz

$n$	Ref. 7 10 elements (60 DOF)	10 elements (10 DOF)	Ref. 2 20 elements (10 DOF)
2	42.9	43.9	42.3
3	24.0	25.7	24.0
4	18.6	20.4	19.0
5	21.1	22.3	21.5

## References

- <sup>1</sup>Bismarck-Nasr, M. N. and Costa-Savio, H. R., "Finite-Element Solution of the Supersonic Flutter of Conical Shells," *AIAA Journal*, Vol. 17, Oct. 1979, pp. 1148-1150.
- <sup>2</sup>Ueda, T. and Kobayashi, S., "Supersonic Flutter of Conical Shells, I: Linear Analysis," *Journal of the Japan Society for Aeronautical and Space Sciences*, Vol. 25, No. 276, Jan. 1977, pp. 40-47 (in Japanese).
- <sup>3</sup>Ueda, T., Kihira, M., and Kobayashi, S., "Supersonic Flutter of Conical Shells, II: Wind Tunnel Tests," *Journal of the Japan Society for Aeronautical and Space Sciences*, Vol. 25, No. 277, Feb. 1977, pp. 95-101 (in Japanese).
- <sup>4</sup>Ueda, T., Kobayashi, S., and Kihira, M., "Supersonic Flutter of Truncated Conical Shells," *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 20, No. 47, 1977, pp. 13-30.
- <sup>5</sup>Ueda, T., "Nonlinear Analysis of the Supersonic Flutter of a Truncated Conical Shell Using the Finite Element Method," *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 20, No. 50, 1978, pp. 225-240.

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<sup>6</sup>Dixon, S. C. and Hudson, M. L., "Flutter, Vibration and Buckling of Truncated Orthotropic Conical Shells with Generalized Elastic Edge Restraint," NASA TN D-5759, 1970.

<sup>7</sup>Adelman, H. M., Catherines, D. S., and Walton, W. C., Jr., "A Method for Computation of Vibration Modes and Frequencies of Orthotropic Thin Shells of Revolution Having General Meridional Curvature," NASA TN D-4972, 1969.

**Table 1 Natural frequencies of a 120 deg conical frustum, Hz**

n	Ref. 2	Present analysis		
	10 elements	6 elements	12 elements	20 elements
1	96.87	100.35	96.83	95.97
2	42.87	47.51	43.18	42.21
3	23.96	29.78	24.91	23.89
4	18.62	24.13	19.70	18.85
5	21.14	24.80	21.79	21.33
6	26.88	29.00	27.21	26.98

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## Reply by Authors to T. Ueda and S. Kobayashi

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**W**E wish to thank T. Ueda and S. Kobayashi for their interesting comments and for their interest in our work. The object of this reply is to clarify some points they have raised.

1) Due to space limitation only a few of the cases studied were reported in our Note.<sup>1</sup> However, more than ten cases were studied in a comprehensive way, including the 120 deg cone free-vibration analysis reported by Ueda and Kobayashi.

Our results for this case are reported in Table 1 and are compared with those of Ref. 2 where the geometric and material data of the shell are given.

2) For the flutter case studied, Novozhilov's shell theory shows that the simplified Mushtari-Donnell theory used by the commenters is in error by 5-10% in the frequencies calculated and it may be greater for other shell geometry and  $n$  as stated in our Note.

3) The neglect of the in-plane inertias can lead to errors up to 40% in the frequencies calculations, e. g., see Ref. 3.

4) The commenters claim to use a static condensation for the flutter problem. However, this cannot be accepted as a rule of thumb, especially for the complicated problem treated here, since it lacks rationalization. Further, it depends on intuition and can lead to erroneous results, as the commenters themselves state in their work.<sup>4</sup>

## References

<sup>1</sup>Bismarck-Nasr, M. N. and Costa Savio, H. R., "Finite-Element Solution of the Supersonic Flutter of Conical Shells," *AIAA Journal*, Vol. 17, Oct. 1979, pp. 1148-1150.

<sup>2</sup>Adelman, H. M., Catherines, D. S., and Walton, W. C. Jr., "A Method for Computation of Vibration Modes and Frequencies of Orthotropic Thin Shells of Revolution Having General Meridional Curvature," NASA TN D-4972, 1969.

<sup>3</sup>Leissa, A. W., "Vibrations of Shells," NASA SP-288, 1973.

<sup>4</sup>Ueda, T., Kobayashi, S., and Kihira, M., "Supersonic Flutter of Truncated Conical Shells," *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 20, No. 47, 1977, pp. 13-30.

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